Solving Linear Equations

Isolating the Variable

Integrating PEMDAS

Word Problems

General Rules in Equations

- The presence of an equal sign is the hallmark of an equation
- The key to solving any equation is to isolate the variable on one side and get a plain number (on the other)
- A single equation can only be solved if it has only a single variable
- ***Key Rule: You can do any mathematical operation to one side of an equation as long as you to the same to the other side!***

Using Inverse Functions to Isolate Variables

- You can generally isolate a variable by getting rid of whatever terms are with it.
  - If the variable is added to a term, subtract it
    - x+3=7; subtract 3 from each side, to get x=4
    - y-8=-10; add 8 to each side to get y=-2
  - If the variable is multiplied by a term, divide each side by that term
    - 4x=44; divide each side by 4 to get x=11
  - If the variable is divided by a term, multiply each side by that term
    - (1/3)y = 8; multiply each term by 3 to get y=24
    - (2/5)x=16 multiple each term by 5 AND divide each term by 2 to get x = 40

Equations with more than one Operation

- Example: $3x + 7 = 22$
- First you need to "get rid" of the terms distinct from the variable.
  - Get rid of the 7 by subtracting 7 from each side
  - (subtracting 7 from each side gets you $3x=15$)
- Then use multiplication or division to finish it off.
  - $3x=15$; divide each side by 3
- Answer: $x=5$

Checking Your Solution

- To check a solution to an equation, simply plug in your answer for the variable in the original equation and see if the mathematical statement is true.
- Example:
  - $4y - 17 = 63$
  - Add 17 to each side to get $4y=80$, which means $y=20$
- Check
  - $4y-17=63$
  - $4(20)-17=63$
  - $80-17=63$
  - $63=63$ (correct!)

Using the Least Common Denominator

- Equations are much easier without fractions.
- So, you can get rid of all denominators by multiplying each expression by the "LCD"
- LCD = the lowest value that can divide into each denominator yielding a whole number
- Example: $x/4 + 1/3 = 20/6$. The LCD is 12
- Multiple each term by 12:
  - $12(x/4) = 3x$; $12(1/3) = 4$; $12(20/6) = 40$
  - $3x+4=40$ → $3x = 36$ → $\underline{x = 12}$

Distributing to Open Parentheses

- Equations can best be solved with no parentheses, so opening parentheses is the first step.
- $3(x+7) = 30$
- $3x + 21 = 30$
- $3x = 9$
- <u>$x = 3$</u>

Variables on Both Side

- If you have a variable term on each side, you need to move all the variables to one side and the numbers to the other to isolate the variable
- 5(y+7) = 7y + 5
  - First, open the parentheses:
  - 5y + 35 = 7y + 15
  - Now, you can either move the variables to the left and numbers to the right by subtracting 7y and 35 from each side: -2y = -20; <u>y = 20</u>
  - Or, you can either move the variables to the right and numbers to the left by subtracting 5y and 55 from each side: 2y = 20; <u>y = 20</u>
    - Note: Ultimately, whether the variable is on the left or right makes no difference.

Word Problems

- The key to doing a word problem is to reduce an English statement to a mathematical expression. Some key terms:
  - Sum = addition
  - Difference = subtraction
  - Times or sets or product = multiplication
    - (e.g., 3 sets of 4)
  - Quotient = division
  - “is” = equals

Word Problems – Let’s Statements

- A “let’s statement” sets a certain symbol (e.g., an x) are the variable you’re looking for.
- In simple word problems, these hardly seem necessary, but when the variable is subject to different factors, the let’s statements become critical.
- Simple example:
  - The difference between 4 times a number and 7 is 21. Find the number.
  - “Let x = the number”
    - $4x - 7 = 21$
    - $x = 7$; so the number is 7

Word Problems – More Complex

- Jane is 2 years older than her brother, Bob. When Jane is twice as old as she is now, she will be 10 years older than Bob is now. Find their current ages.
  - We ***need*** both siblings' ages to be quantified by the same variable
    - Let x = Bob's current age
    - Let x+2 = Jane's current age (x-2 for Bob and x for Jane also works)
  - Now, set up the equation:
    - When Jane is twice as old as now: 2(x+2)
    - We also know Jane's age then is x + 10
  - So, the equation is: $2(x+2) = x + 10$
    - $2x + 4 = x + 10$ gives you $\underline{x = 6}$
  - Plugging x into our let's statements gives us Bob's current age as 6 and Jane's current age as 8.